1857MNRAS..17..155D

Table II.—*Inégalités du Logarithme du Rayon Vecteur du Soleil.*

Arg. V. + Arg. XIII. + le Jour de l'Année.

Arg.	Inégalité à Mult. p. $\left(\frac{t-1850}{100}\right)^3$	Diff.	Inégalité à Mult. p. $\left(\frac{t-1850}{100}\right)^4$	Arg.	Inégalité à Mult. p. $\left(\frac{t-1850}{100}\right)^3$	Diff.	Inégalité à Mult. p. $\left(\frac{t-1850}{100}\right)$
0	−0·026		−0·00002	192	+0·009		−0·00005
8	−0·014	+12	−0·00002	200	−0·003	−12	−0·00004
16	−0·001	13	−0·00002	208	−0·014	11	−0·00004
24	+0·011	12	−0·00002	216	−0·025	11	−0·00004
32	+0·023	12	−0·00002	224	−0·035	10	−0·00003
40	+0·035	12	−0·00002	232	−0·045	10	−0·00002
48	+0·046	11	−0·00002	240	−0·055	10	−0·00002
56	+0·055	9	−0·00002	248	−0·063	8	−0·00001
64	+0·064	9	−0·00002	256	−0·070	7	0·00000
72	+0·072	8	−0·00002	264	−0·076	6	0·00000
80	+0·078	6	−0·00001	272	−0·081	5	+0·00001
88	+0·082	4	−0·00001	280	−0·084	3	+0·00001
96	+0·085	3	−0·00001	288	−0·086	−2	+0·00001
104	+0·086	+1	−0·00002	296	−0·086	0	+0·00001
112	+0·085	−1	−0·00002	304	−0·085	+1	+0·00001
120	+0·083	2	−0·00002	312	−0·082	3	+0·00001
128	+0·079	4	−0·00002	320	−0·077	5	+0·00001
136	+0·074	5	−0·00003	328	−0·071	6	0·00000
144	+0·067	7	−0·00003	336	−0·063	8	0·00000
152	+0·059	8	−0·00004	344	−0·054	9	−0·00001
160	+0·051	8	−0·00004	352	−0·044	10	−0·00001
168	+0·041	10	−0·00005	360	−0·033	11	−0·00002
176	+0·031	10	−0·00005	368	−0·022	11	−0·00002
184	+0·020	11	−0·00005	376	−0·010	12	−0·00002
192	+0·009	−11	−0·00005	384	+0·003	+13	−0·00002

On a Numerical Solution of Kepler's Problem, reduced to a Tabular Form. By Dr. A. De Gasparis.

(*Extracted from Letters addressed to Mr. Hind.*)

"I have constructed a numerical table for the solution of Kepler's problem. This table, which extends to 23 pages, similar

1857MNRAS..17..155D

in size to those of Callet, gives, by simple inspection, the value of the excentric anomaly within about a degree, for any excentricity less than unity. By proportional parts we get the value of our unknown quantity to a fraction of a minute, and then by a calculation of the correction $\Delta\varepsilon$ by the formula

$$\Delta\varepsilon = \frac{\mathrm{M} - \varepsilon + e \sin \varepsilon}{1 - e \cos \varepsilon}$$

we get it within a fraction of a second.

"The table, consequently, contains all possible solutions of the equation

$$\mathrm{M} = \varepsilon - e \,.\, \sin \varepsilon$$

for whole-degree values of M and ε, and for values of e less than 1. It would not be difficult to obtain these solutions directly, but the calculation would be long, and there is the objection that we cannot beforehand estimate its extent. Allow me to explain the process I have followed.

"I remarked that if we wish to solve practically (say by a mechanical process) the equation

$$m = x \pm n \,.\, \varphi\,(x)$$

in which the function φ is one of those which has been tabulated, we must mark on a rule the logarithms of the numbers c_1, c_2, c_3, &c., found by the equations

$$h = c_1\,\varphi\,(h) = c_2\,\varphi\,(2\,h) = c_3\,\varphi\,(3\,h), \text{ \&c.}$$

and place above them the numbers h, $2\,h$, $3\,h$, &c. The number h is arbitrary. This done, we must mark on another rule either from right to left, or from left to right, the logarithms of the numbers 0, 1, 2, 3, &c. If, now, we want to find the roots of the equation

$$m = x \pm n\,\varphi\,(x)$$

we must find on the first rule a number $p\,h$ nearly equal to m; place it beneath the $-\infty$ of the second rule, and take at sight the sums of the logarithms of c_n of the first rule and the corresponding logarithms of the natural numbers on the second rule. The sum which gives the logarithm of n will be found at the part of the rule under the number $q\,h$; and this number will be the approximate root. We may correct this value by the proportions which are found of themselves by placing the symbol $-\infty$ first under the number next less than m, and, secondly, under the number next greater than m.

"In the case of Kepler's problem, I have taken $h = 1°$. I have then deduced the solutions which I wanted by the simple addition of the logarithms which are found at corresponding points on the two rules; and I have stopped the calculations in all cases where the logarithm of the sum exceeded zero.

"The following table is equivalent to two-fifths of the twenty-three pages of which the whole table is composed. In the example

1857MNRAS..17..155D

subjoined, after the calculation of proportional parts, I find the correction by means of the formula $\Delta\varepsilon = \frac{M - \varepsilon + e \sin \varepsilon}{1 - e \cos \varepsilon}$.

Specimen of Table.

M	e	ε	M	e	ε	M	e	ε	M	e	ε
°		°	°		°	°		°	°		°
26	0·36054	39	26	0·80389	69	27	0·40946	43	27	0·83954	73
26	0·38014	40	26	0·81723	70	27	0·42713	44	27	0·85536	74
26	0·39905	41	26	0·83066	71	27	0·44429	45	27	0·86731	75
26	0·41734	42	26	0·84417	72	27	0·46100	46	27	0·88139	76
26	0·43506	43	26	0·85779	73	27	0·47729	47	27	0·89562	77
26	0·45225	44	26	0·87152	74	27	0·49320	48	27	0·91001	78
26	0·46897	45	26	0·88538	75	27	0·50877	49	27	0·92456	79
26	0·48526	46	26	0·89938	76	27	0·52403	50	27	0·93930	80
26	0·50115	47	26	0·91353	77	27	0·53900	51	27	0·95423	81
26	0·51669	48	26	0·92785	78	27	0·55372	52	27	0·96937	82
26	0·53189	49	26	0·94234	79	27	0·56820	53	27	0·98473	83
26	0·54681	50	26	0·95702	80	27	0·58248	54	27	1·00032	84
26	0·56146	51	26	0·97190	81	27	0·59658	55	28	0·03600	29
26	0·57586	52	26	0·98699	82	27	0·61052	56	28	0·06981	30
26	0·59006	53	26	1·00231	83	27	0·62432	57	28	0·10166	31
26	0·60406	54	27	0·03718	28	27	0·63800	58	28	0·13174	32
26	0·61789	55	27	0·07200	29	27	0·65157	59	28	0·16023	33
26	0·63158	56	27	0·10472	30	27	0·66506	60	28	0·18727	34
26	0·64513	57	27	0·13555	31	27	0·67848	61	28	0·21300	35
26	0·65858	58	27	0·16468	32	27	0·69185	62	28	0·23755	36
26	0·67193	59	27	0·19227	33	27	0·70518	63	28	0·26101	37
26	0·68521	60	27	0·21848	34	27	0·71849	64	28	0·28349	38
26	0·69844	61	27	0·24343	35	27	0·73179	65	28	0·30507	39
26	0·71162	62	27	0·26724	36	27	0·74510	66	28	0·32583	40
26	0·72477	63	27	0·29001	37	27	0·75842	67	28	0·34584	41
26	0·73791	64	27	0·31184	38	27	0·77178	68	28	0·36517	42
26	0·75105	65	27	0·33280	39	27	0·78519	69	28	0·38387	43
26	0·76420	66	27	0·35298	40	27	0·79866	70	28	0·40200	44
26	0·77739	67	27	0·37245	41	27	0·81220	71	28	0·41961	45
26	0·79061	68	27	0·39125	42	27	0·82582	72	28	0·43673	46

Example.—Let the equation be $26^\circ{\cdot}35794 = \varepsilon - 0{\cdot}82575 \sin \varepsilon$.

For $M = 26^\circ$, and by variation of e we find $\varepsilon = 70^\circ{\cdot}63440$.

For $M = 27^\circ$, and by variation of e we find $\varepsilon = 71^\circ{\cdot}99486$.

Whence for $M = 26^\circ{\cdot}35794$, we shall have $\varepsilon = 71^\circ{\cdot}12136$.

The error is $0^\circ{\cdot}003$. The correction $\Delta\,\varepsilon$ comes out $+\ 0{\cdot}00472$."

Whence, lastly, $\varepsilon = 71^\circ{\cdot}12608$. Error $= 0^\circ{\cdot}000003$.

An Expeditious Method of finding the Latitude by simply observing the Azimuthal Angles of two Circumpolar Stars with a Theodolite. By M. Babinet.

(*Communicated by Admiral W. H. Smyth.*)

It is presumed that the method is designed to meet the case of a traveller, who, from other considerations, has been led to take with him a small theodolite and pocket-chronometer, to the exclusion of a sextant or repeating circle; and that he has with him a catalogue of the brighter stars. Regarded otherwise, the method would be much inferior to others already known.

In the well-known triangle Z P S, let Z be the zenith, P the pole, and S the place of a circumpolar star. $ZP = 90 - \lambda$. $PS = \delta$. $PZS = A$, the azimuth of the star.

Then

$$\sin A \,.\, \cos \lambda = \sin \delta \,.\, \sin PSZ$$

It is proposed that the star's azimuth shall be observed when at its greatest elongation, in which case $PSZ = 90^\circ$, and the above equation becomes

$$\sin A \,.\, \cos \lambda = \sin \delta \qquad (1)$$

Similarly for the second star we should have

$$\sin A' \,.\, \cos \lambda = \sin \delta' \qquad (2)$$

Dividing (1) — (2) by (1) + (2), we then have

$$\frac{\tan \frac{1}{2}(A - A')}{\tan \frac{1}{2}(A + A')} = \frac{\tan \frac{1}{2}(\delta - \delta')}{\tan \frac{1}{2}(\delta + \delta')} \qquad (3)$$

When the two stars are observed at opposite elongations, A and A′ will have opposite signs, and $A + A'$ will be found at once from the difference of the two readings of the horizontal circle, half the difference will be

$$\tfrac{1}{2}(A + A')$$

By equation (3) we thence get

$$\tfrac{1}{2}(A - A')$$

Thence A and A′ separately, and by either (1) or (2) the required latitude λ.

Having found from A or A′ the azimuthal reading of the pole, we can then set the telescope in the meridian, and obtain the time very nearly by a transit of an intertropical star. The whole operation need not take more than twenty minutes, if the stars are previously suitably selected.